ANALYSIS OF DIGITAL SEARCH TREES BUILT ON A GENERAL SOURCE

Kanal Hun and Brigitte Vallée,

GREYC (CNRS and University of Caen)

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This talk : Analysis of Digital Search Trees

when they are built on words emitted by a general source.

I. Trees.

Description of the DST structure (I)

On the alphabet $\Sigma := \{a, b\}$, consider the sequence of six words

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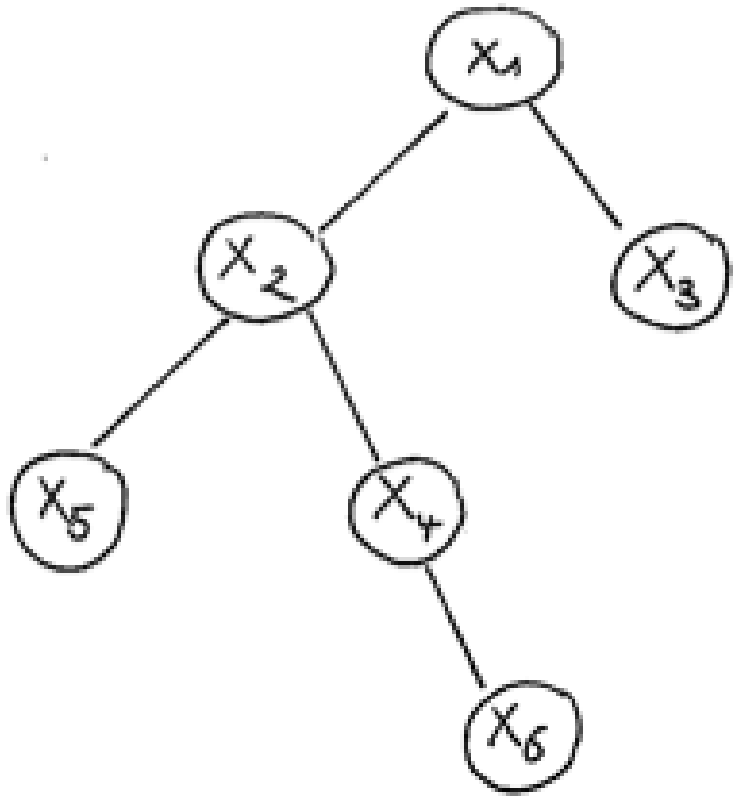

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The three tree structures and their (internal or external) path length

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DST Trie Symbol-BST

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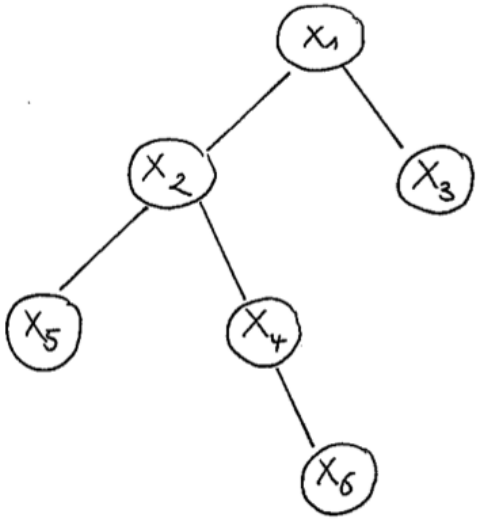
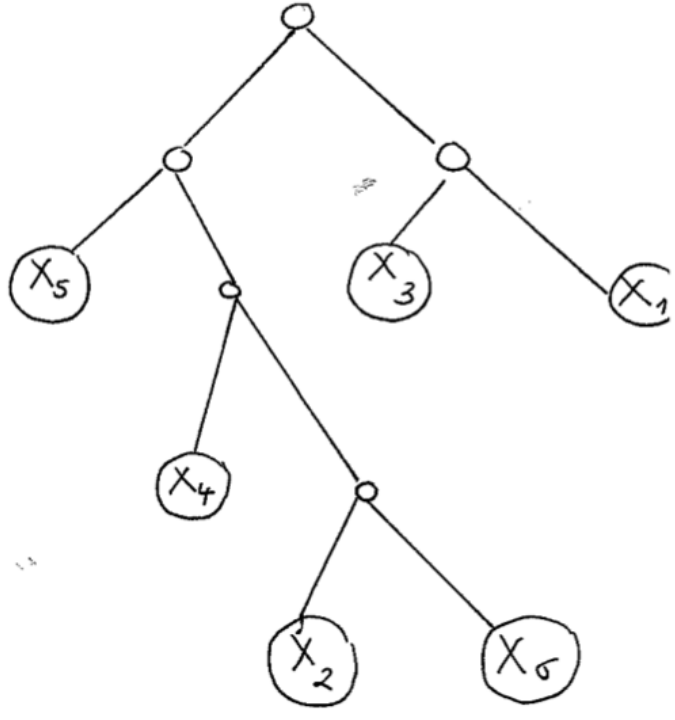
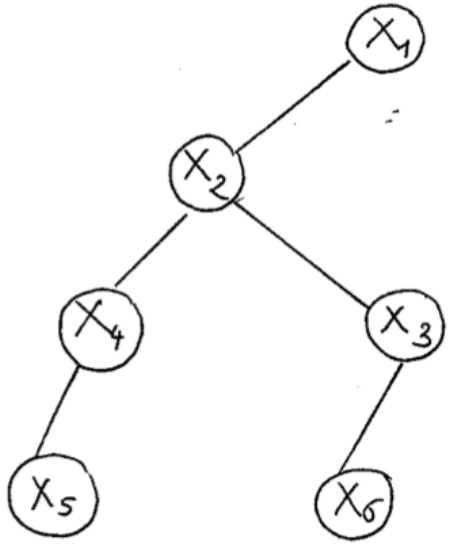


Trie



Symbol-BST



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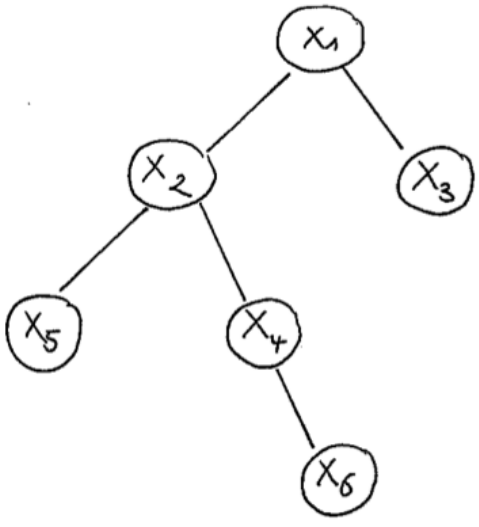
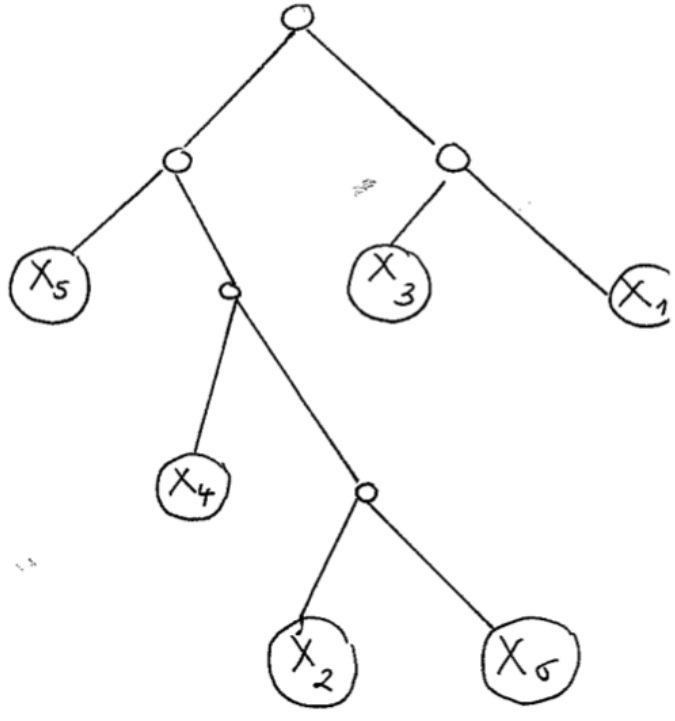


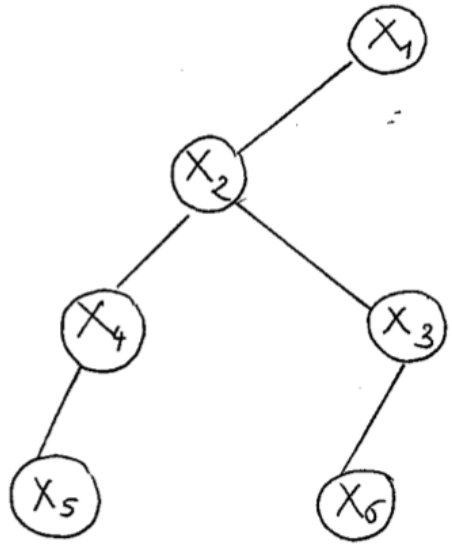
$2 + 2 \times 2 + 1 \times 3$
$= 9$

Trie



$3 \times 2 + 1 \times 3 + 2 \times 4$
$= 17$

Symbol-BST



$1 + 3 + 4 + 5 + 6$
$= 19$

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built on n infinite words independently emitted from the same source $\mathcal{S}$

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- The full nodes for Tries: external nodes

The depth of a node: the number of nodes between the node and the root.

Main parameters of interest

- the path length L_n := the sum of the depths of the full nodes
- the profile $B_{n,k}$:= number of full nodes at depth k
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For applications, importance to deal with a general source $\mathcal{S}$

Results already obtained for DST in the case of simple sources.

To be compared to Tries and BST (results obtained for general sources)

A source:= a random mechanism which emits symbols from alphabet Σ,

$$\text{The time is discrete } t = 0, t = 1, \ldots$$
$$X_i := \text{the symbol emitted at time } t = i$$

When time evolves, the source produces (infinite) words of $\Sigma^{\mathbb{N}}$.

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A general source may have many, strong correlations between its symbols.
For $w \in \Sigma^\star$, $p_w :=$ probability that a word begins with the prefix w.
The set $\{p_w, \quad w \in \Sigma^\star\}$ defines the source $\mathcal{S}$.

Analyses of Tries and Symbol–BST built on general sources

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(i) The mean path-length of the Trie satisfies $$L_n^{[T]} \sim \frac{1}{h_{\mathcal{S}}}\, n\, \log n.$$

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The remainder term depends on the "tameness" region of the source.

Previous results for a DST built on a simple source

Simple source = memoryless source or ergodic Markov chain.

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The difference $A_{\mathcal{S}} - B_{\mathcal{S}}$ is always negative for any simple source.

The remainder terms R_n are of the same type for the two structures,

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For the binary unbiased source: $\quad A_{\mathcal{S}} - B_{\mathcal{S}} = -\left[\frac{1}{\log 2} + \sum_{k\geq 1} \frac{1}{2^k - 1}\right]$

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Consider a super-tame source $\mathcal{S}$. Then, the mean internal path-length of a digital search tree built on n words independently emitted by $\mathcal{S}$ satisfies

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$$L_n^{[T]} = \frac{1}{h_{\mathcal{S}}} n \log n + B_{\mathcal{S}}\, n + R_n^{[T]}$$

We obtain an expression for $A_{\mathcal{S}} - B_{\mathcal{S}}$ which proves that $A_{\mathcal{S}} < B_{\mathcal{S}}$

II. Sources.

A main analytical object related to any source:
the Dirichlet generating functions of the source

$$\Lambda(s) := \sum_{w \in \Sigma^\star} p_w^s, \qquad \Lambda_k(s) = \sum_{w \in \Sigma^k} p_w^s, \qquad \left[\Lambda = \sum_{k \geq 0} \Lambda_k \right]$$

Remark: $\Lambda_k(1) = 1$ for any k, $\Lambda(1) = \infty$.

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– they intervene in probabilistic analysis of algorithms and data structures.

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Markov chains, defined by – the vector $\mathbf{R}$ of initial probabilities (r_i)

– and the transition matrix $\mathbf{P} := (p_{j|i})$

$$\Lambda(s) = 1 + {}^t\mathbf{R}_s (I - \mathbf{P}_s)^{-1} \mathbf{1} \qquad \text{with} \quad \mathbf{P}_s = (p_{j|i}^s), \quad \mathbf{R}_s = (r_i^s).$$

A main analytical object related to any source:

the Dirichlet series of probabilities, $\Lambda(s) := \sum_{w \in \Sigma^\star} p_w^s$

Memoryless sources, with probabilities (p_i)

$$\Lambda(s) = \frac{1}{1 - \lambda(s)} \qquad \text{with} \quad \lambda(s) = \sum_{i=1}^{r} p_i^s$$

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And for a general source?

Does $\Lambda(s)$ admit a nice alternative expression?

A general source $\mathcal{S}$ and its shifted sources.

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For $u \in \Sigma^\star$ with $p_u \neq 0$, the source $\mathcal{S}_{(u)} = \mathcal{S}|_u$ is a shifted source

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In this case, w can be written as $w = u \cdot v$

The conditional probabilities $p_{w|u} = p_{(u.v)|u}$ are denoted as $q_{v|u}$.

These are the fundamental probabilities of the source $S_{(u)}$.

The generalized transition matrix of a source $\mathcal{S}$

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extends the transition matrix of a Markov chain.

This is an infinite matrix, whose rows and columns are indexed by $\Sigma^\star$

The non zero coefficients at the row u are located at the columns $u.i$

they are equal to: $$\mathbf{P}(u, u \cdot i) = \frac{p_{u \cdot i}}{p_u} = q_{i|u}$$

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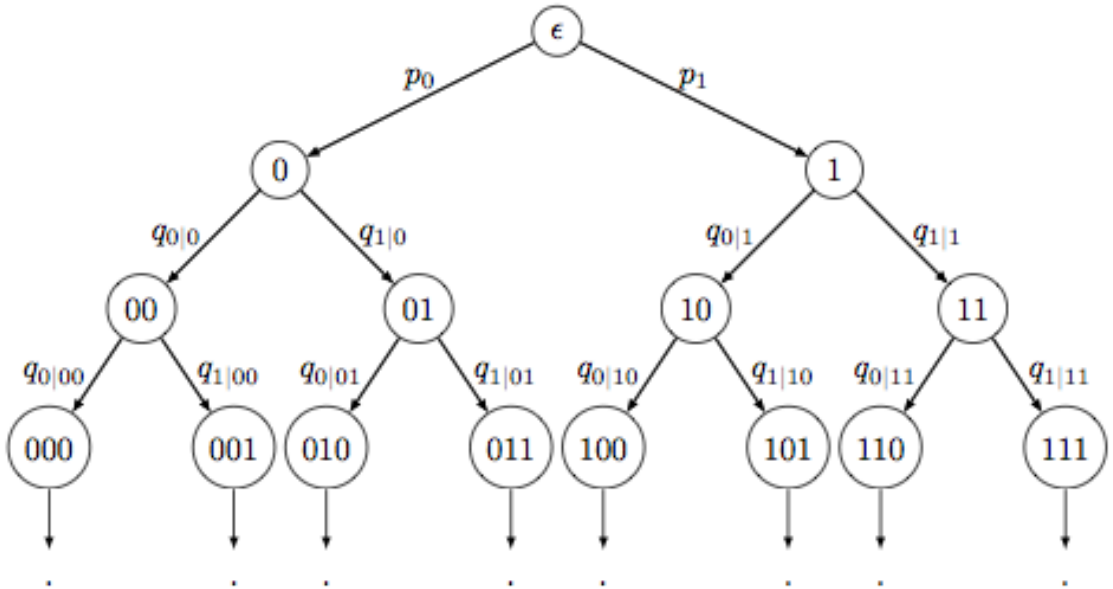
The matrix $\mathbf{P}$ is the transition matrix associated to the graph of the source.

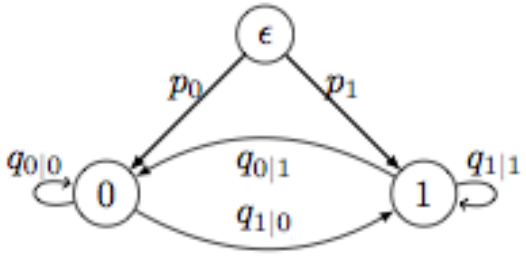
The states are the sources $\mathcal{S}_{(u)}$, the transitions are $u \to u \cdot i$.

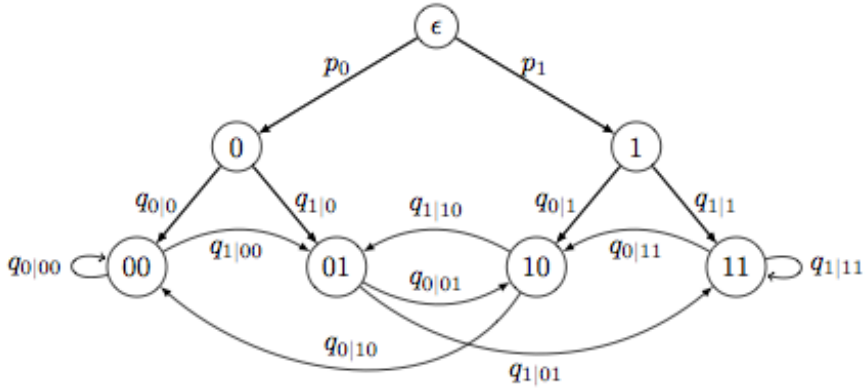


Sometimes, the graph (and thus the matrix) can be pruned:

– One only keeps the sources $S_{(u)}$ which have a different distribution

– For a simple source, this provides a finite graph (a finite matrix)





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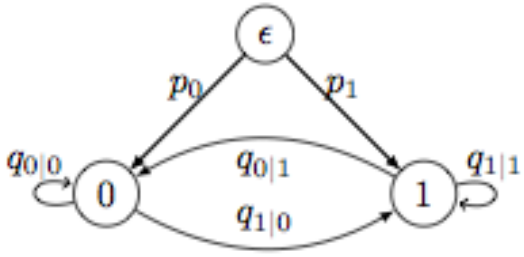

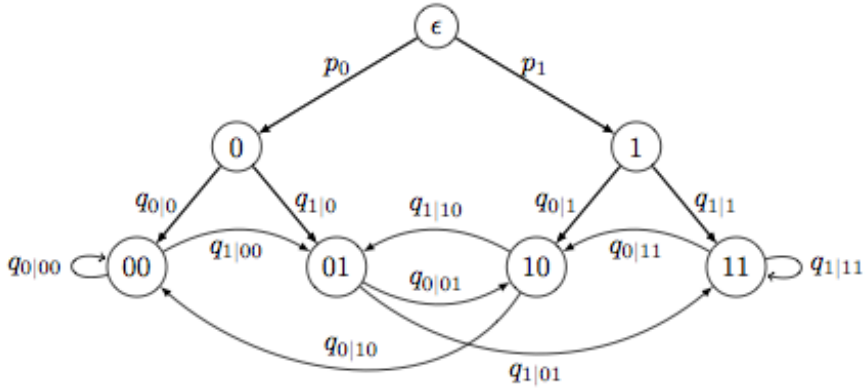

For $s \in \mathbb{C}$, the matrix $\mathbf{P}_s$ is obtained from $\mathbf{P}$

by raising its coefficients to the power s.

For $k \in \mathbb{N}$, the non zero coefficients of matrix $\mathbf{P}_s^k$ at the row u

are located at the columns $u.\alpha$ (for $\alpha \in \Sigma^k$) and equal $p^s_{u.\alpha|u} = q^s_{\alpha|u}$.

For a general source, with its generalized transition matrix $\mathbf{P}_s$, one has

$$\Lambda(s) = {}^t\mathbf{E} \cdot (I - \mathbf{P}_s)^{-1}[\mathbf{1}], \qquad \text{with} \qquad {}^t\mathbf{E} := (1, 0, 0 \ldots)$$

III. Analyses of tree data structures built on general sources.

A common strategy for the analysis of the three types of trees:
Two dictionaries –algebraic and analytic–

Source $\mathcal{S}$,		Mixed Dirichet series		The mean value L_n
its characteristics	(A)	$\varpi(s)$	(B)	of the path length
and	$\Longrightarrow$	depends both	$\Longrightarrow$	of the data structure
the data structure,		on the source		when built
its recursive definition		and the data structure		on n words of $\mathcal{S}$

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An exact formula, from which the asymptotics can be derived.

A common strategy for the analysis of the three types of trees:
Two dictionaries –algebraic and analytic–

Source $\mathcal{S}$, its characteristics and the data structure, its recursive definition	(A) $\Longrightarrow$	Mixed Dirichet series $\varpi(s)$ depends both on the source and the data structure	(B) $\Longrightarrow$	The mean value L_n of the path length of the data structure when built on n words of $\mathcal{S}$

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(B) $$L_n = \sum_{k=2}^{n} (-1)^k \binom{n}{k} \varpi(k)$$

An exact formula, from which the asymptotics can be derived.
With the Rice formula, it depends on "tameness" properties of $\varpi(s)$
closely related to tameness properties of the source

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$$\varpi_T(s) = s\Lambda(s), \qquad \varpi_B(s) = \frac{1}{s(s-1)}\Lambda(s),$$

Remind: $\Lambda(s) = {}^t\mathbf{E}\,(I - \mathbf{P}_s)^{-1}[\mathbf{1}]$, with ${}^t\mathbf{E} := (1, 0, 0\ldots)$.

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Tameness of the source:

Existence of a region $\mathcal{R}$ on the left of the vertical line $\Re s = 1$ where $\Lambda(s)$

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Then Rice's formula may be applied to $\varpi(s)$.

This gives the previously known results on Tries and Symbol–BST's

Mixed Dirichlet series $\varpi(s)$ of tree structures – (II) DSTs (new...)

For DST's, we prove that the series $\varpi(s)$ is also expressed with $(I-\mathbf{P}_s)^{-1}$,
more precisely with the infinite product

$$\mathbf{Q}_s := (I-\mathbf{P}_s)^{-1}(I-\mathbf{P}_{s+1})^{-1}\dots(I-\mathbf{P}_{s+k})^{-1}\dots,$$

under the form $\varpi_D(s) = {}^t\mathbf{E}\,\mathbf{Q}_s\,\mathbf{Q}_2^{-1}[\mathbf{1}] = {}^t\mathbf{E}(I-\mathbf{P}_s)^{-1}\mathbf{Q}_{s+1}\mathbf{Q}_2^{-1}[\mathbf{1}]$

Finally: $\varpi_D(s) = \Lambda(s) + A(s)$, with

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Then Rice's formula may be applied to $\varpi(s)$ and the theorem is proven.

IV. Some steps of the proof.

We consider a DST built on n words independently emitted by a source $\mathcal{S}$.
We study the random variable path length $\ell_n := \ell_n(\mathcal{S})$, and deal with all the sources $\mathcal{S}_{(w)}$ and all the random variables $\ell_n^{(w)} := \ell_n(\mathcal{S}_{(w)})$

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For $\Sigma := \{0, 1\}$, the basic recurrence for the sequence $\ell_n^{(w)}$ is

$$\ell_n{}^{(w)} = n - 1 + \ell_{K_n}{}^{(w.0)} + P_{n-1-K_n}{}^{(w.1)}$$

The number of nodes $K_n := K_n^{(w)}$ in the left subtree

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$$\Pr[K_n{}^{(w)} = k] = \binom{n-1}{k} q_{0|w}^k \, q_{1|w}^{n-1-k}$$

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Then, the basic recurrence for the expectations $L_n{}^{(w)} := \mathbb{E}[\ell_n^{(w)}]$ is

$$L_n{}^{(w)} = n - 1 + \sum_{k=0}^{n-1} \binom{n-1}{k} q_{0|w}^k \, q_{1|w}^{n-1-k} \left(L_k^{(w\cdot 0)} + L_{n-1-k}^{(w\cdot 1)} \right)$$

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The Poisson generating functions $B^{(w)}(z) := e^{-z} \sum_{n\geq 0} L_n^{(w)} \frac{z^n}{n!}$ satisfy

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Easy extension to any alphabet Σ:

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This system of functional equations involves both

- the derivation d/dz
- the change of variables $z \mapsto qz$
- the shift on words $w \mapsto w.i$

In comparison, for tries, the derivation does not occur.

Different possible steps of the analysis

Basic equations $$\frac{d}{dz}B^{(w)}(z) + B^{(w)}(z) = z + \sum_{i\in\Sigma} B^{(w.i)}(q_{i|w}z).$$

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Exact expression of $B(z)$ $\quad \Longrightarrow$ AlgDePo $\Longrightarrow$ Exact expression of L_n

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$$B(z) = \sum_{w\in\Sigma^\star} \delta(v)[e^{-zp_v} - 1 + zp_v] \qquad L_n = \sum_{k=2}^{n} (-1)^k \binom{n}{k} \varpi_D(k)$$

$$\delta(v) = \frac{1}{p_v} \sum_{w\geq v} p_w \prod_{\substack{\alpha\leq w,\\ \alpha\neq v}} \frac{1}{1 - p_v p_\alpha^{-1}} \qquad \varpi_D(s) = \sum_{w\in\Sigma^\star} \delta(v) p_v^s,$$

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Alternative expression of $\varpi(s)$ $\qquad$ Asymptotics of L_n

$\varpi_D(s) = {}^t\mathbf{E}\,\mathbf{Q}_s\,\mathbf{Q}_2^{-1}[\mathbf{1}]$ $\qquad$ with tameness of $(I - \mathbf{P}_s)^{-1}$

Various extensions

Various extensions

– Already studied : the distribution of the typical depth.

Main result. For a source assumed to be hyper-tame and log–convex, the typical depth of a DST follows an asymptotic gaussian law.

The asymptotic mean values of the mean and the variance involve the same constants as in the case of tries.

Various extensions

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    Work in progress with E. Cesaratto, J. Clément.